Check for updates

RESEARCH ARTICLE

REVISED Effects of variable viscosity, concentration and heat variation on MHD oscillatory flow for Bingham fluid through an inclined porous channel

[version 2; peer review: 2 approved, 2 approved with reservations]

Dheia G. Salih Al-Khafajy[1], Amal Al-Hanaya[2], Munirah Aali Alotaibi[2]

[1]Department of Mathematics, University of Al-Qadisiyah, Diwaniya, Iraq
[2]Mathematical Sciences, College of Science, Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia

v2 **First published:** 06 Apr 2026, **15**:476
https://doi.org/10.12688/f1000research.172909.1
Latest published: 23 Jun 2026, **15**:476
https://doi.org/10.12688/f1000research.172909.2

Abstract

Background

Magnetohydrodynamic oscillatory flows of non-Newtonian fluids in porous channels arise in many industrial and geophysical applications. Understanding the combined influence of variable viscosity, heat generation, and concentration is essential for accurate flow prediction.

Methods

A mathematical model for unsteady MHD oscillatory flow of a Bingham fluid through an inclined porous channel was formulated. The governing nonlinear partial differential equations for momentum, energy, and concentration were nondimensionalized and solved using the separation of variables technique. Numerical evaluation and graphical analysis were performed using Wolfram Mathematica.

Results

The results show that increasing heat generation and radiation parameters enhances fluid temperature and velocity, while higher magnetic and oscillation parameters suppress flow motion. Concentration was found to increase with higher oscillation frequency and Péclet number, whereas Schmidt and chemical reaction parameters reduced mass diffusion. Variable viscosity significantly amplified velocity compared to constant-viscosity cases.

Open Peer Review

Approval Status ✓ ? ✓ ?

	1	2	3	4
version 2 (revision) 23 Jun 2026	✓ view		✓ view	? view
version 1 06 Apr 2026	? view	? view		

1. **Oluwole Daniel Makinde**, Stellenbosch University, Cape Town, South Africa
2. **Gabriella Bognár**, University of Miskolc, Miskolc-Egyetemváros, Hungary
3. **Ali J. Chamkha**, Kuwait College of Science and Technology, Doha, Kuwait
4. **Lakshminarayana P**, Vellore Institute of Technology, Vellore, India

Any reports and responses or comments on the article can be found at the end of the article.

Conclusions

The study demonstrates that temperature-dependent viscosity and yield-stress effects strongly control MHD oscillatory Bingham fluid flow in inclined porous channels. The results are relevant to engineering systems involving non-Newtonian transport with thermal and mass diffusion effects. The novelty of the present formulation lies in combining Bingham yield-stress behavior, temperature-dependent viscosity, porous-medium resistance, magnetic-field effects, heat generation, radiation, and concentration transport within a single analytical framework.

Keywords
Bingham fluid, thermal transfer, concentration, magnetohydrodynamic (MHD), inclined porous channel

Corresponding author: Munirah Aali Alotaibi (maalotaiby@pnu.edu.sa)

Author roles: Al-Khafajy DGS: Conceptualization, Data Curation, Formal Analysis, Methodology, Project Administration, Software, Supervision, Validation, Writing – Original Draft Preparation, Writing – Review & Editing; **Al-Hanaya A**: Conceptualization, Data Curation, Formal Analysis, Methodology, Software, Visualization, Writing – Original Draft Preparation, Writing – Review & Editing; **Alotaibi MA**: Data Curation, Formal Analysis, Funding Acquisition, Methodology, Project Administration, Writing – Original Draft Preparation, Writing – Review & Editing

Competing interests: No competing interests were disclosed.

Grant information: This research was funded by Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2026R522), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.
The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Copyright: © 2026 Al-Khafajy DGS *et al*. This is an open access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

How to cite this article: Al-Khafajy DGS, Al-Hanaya A and Alotaibi MA. **Effects of variable viscosity, concentration and heat variation on MHD oscillatory flow for Bingham fluid through an inclined porous channel [version 2; peer review: 2 approved, 2 approved with reservations]** F1000Research 2026, **15**:476 https://doi.org/10.12688/f1000research.172909.2

First published: 06 Apr 2026, **15**:476 https://doi.org/10.12688/f1000research.172909.1

REVISED **Amendments from Version 1**

In this revised version, the governing formulation has been carefully rechecked and corrected to ensure consistency between the dimensional momentum equation and its corresponding dimensionless form. The abstract and introduction have been updated to state the novelty of the work more clearly, especially the combined treatment of oscillatory MHD mixed convection, Bingham yield-stress behavior, variable viscosity, porous-medium resistance, and coupled heat and mass transfer in an inclined channel. The introduction has also been strengthened by incorporating additional recent and relevant literature to better position the study within the current research context. In addition, the physical interpretation of the results has been improved by discussing the competition between buoyancy forces, Lorentz-force damping, yield-stress resistance, viscosity variation, and porous drag. We have clarified the explanation of the main dimensionless parameters and polished the manuscript to improve readability, consistency, and scientific presentation.

Any further responses from the reviewers can be found at the end of the article

1. Introduction

A Bingham fluid is a category of non-Newtonian fluid that exhibits solid-like behavior at low-stress levels and flows as a viscous liquid under elevated stress levels. They demonstrate yield stress, indicating that the fluid remains static until the applied stress is above a specific critical threshold. Below this yield stress, the material exhibits rigid body behavior. Upon surpassing the yield stress, the flow behavior becomes linear, characterized by a constant viscosity. The linear correlation between stress and strain rate manifests similarly to that observed in Newtonian fluids. Typical instances encompass toothpaste, mayonnaise, and concrete. These materials necessitate a certain force to initiate flow. Bingham fluids are significant in numerous industrial and scientific applications, especially where materials necessitate exact regulation of flow characteristics. Bird et al.[1] examined the behaviour of a Bingham fluid within a hard circular tube. Kapur[2] introduced several mathematical models, such as Cais-son, the Bingham, and Herschel-Bulkley models. Rathy[3] examined the fluid dynamics of a Bingham fluid within a channel and an annulus featuring impermeable barriers. Vajravelu et al.[4] conducted an experimented on the flow of Bingham fluid in an annular tube with a porous wall. Ramakrishna et al[5] investigated the flow behaviour of a Bingham fluid on a permeable bed in an enclosed channel. Goverdhan[6] examined the flow of Bingham fluid in a porous channel. Narahari et al.[7] Investigate the movement of a Bingham fluid between two porous substrates, focusing on its unsteady behaviour. Recently, Murthy et al.[8] focused on analysing the flow of a Bingham fluid, which is not stable, in contact with a Newtonian fluid within two parallel plates. The objective is to determine the velocity field, mass flow rates, and interface velocity. Tsangaris et al.[9] conducted on the movement of a Bingham fluid between two porous walls that are parallel to each other. One wall moves steadily in the same direction as the other wall, which remains stationary. At the same time, there is a pressure difference throughout the length of the wall, and there is a flow of fluid across the walls due to their porous nature. Adnan and Abdulhadi[10] performed an investigation on the influence of an inclined magnetic field on the flow of incompressible Bingham plastic fluid within an inclined symmetric channel. The study also considered the effects of mass transfer and heat transfer. Slip circumstances are utilised for heat transmission and focus. Lakshminarayana et al.[11] examined the impact of wall slip circumstances, elasticity wall characteristics, and heat transfer on the movement of conducting Bingham fluid in an irregular channel using the presumptions of a lengthy wavelength and low Reynolds number. The current research by Mahabaleshwar et al.[12] investigates how radiation and chemical reactions affect the two-dimensional boundary layer flow of a bi-viscous Bingham fluid on a thermosolutal Marangoni boundary that is accompanied by a magnetic field and thermal source or sink. A mathematical model is developed using the Navier-Stokes equations to represent the physical flow problem. In order to convert these nonlinear PDEs into a system of nonlinear ODEs, they employed a similarity transformation. The study conducted by Basavarajappa et al.[13] examines the multilayer flow of a bi-viscous Bingham fluid within a vertical slab with a hybrid nanofluid, using the nonlinear Boussinesq approximation. Recent studies have continued to examine oscillatory and unsteady MHD convection in thermally active media. Afridi et al.[14] analyzed entropy generation minimization and heat-transfer enhancement in MHD oscillatory ferrofluid flow, emphasizing the role of magnetic control in thermal transport. Ullah et al.[15] studied oscillatory mixed convection around a horizontal circular cylinder embedded in a porous medium while considering temperature-dependent density effects. Nandkeolyar et al.[16] investigated unsteady hydromagnetic natural convection of a dusty fluid past a moving vertical plate with ramped temperature and thermal radiation. These studies show that unsteadiness, magnetic effects, buoyancy, and porous-medium resistance remain important mechanisms in convective transport problems.

Heat transport in Bingham fluids entails intricate interplay between the fluid's rheological features and thermal aspects. The existence of a yield stress, which regulates flow commencement, profoundly influences heat transfer processes. Vradis et al.[17] numerically resolved the concurrent evolution of thermal fields and hydrodynamic at the entry region of a cylindrical pipe for a non-Newtonian Bingham-type fluid by employing the completely elliptic mathematical models of continuity, momentum, and energy. Mustafa et al.[18] conducted a heat transfer simulation for the swirling flow of a Bingham fluid constrained by a permeable rotation disk. The influence of concentration and temperature variations on

magnetohydrodynamic (MHD) oscillatory flow in a porous media has numerous practical implications in the fields of engineering, industry, medical research, and issues related to the extraction and transportation of petroleum. Hamza et al.[19] conducted a study on the impact of slip condition, radiative heat transfer, and transverse magnetic field on the unsteady flow of a conducting optical thin fluid via a channel equipped with a porous media. Khudair and Al-Khafajy[20] proposed a heat transfer model for MHD oscillating flow of Williamson fluid over a porous plate, considering two different forms of flow. Al-Aridhee and Al-Khafajy[21] examined the impact of mass and heat transfer on the peristaltic movement of MHD flow of a non-Newtonian Jeffrey fluid via a cylindrical porous media channel. The investigation focuses on the flow within a wave frame of reference that is travelling at the velocity of the wave. Al-Khafajy and Labban[22] conducted a study on the combined impact of concentration and thermo-diffusion on the fluctuating flow of an incompressible Carreau fluid via an angled permeable channel. Liu et al.[23] investigate the steady flow and thermal transfer of Bingham fluid over a spinning disk of limited radius with radially varying thickness in the boundary layer. Eldabe et al.[24] examined the flow of non-Newtonian Bingham blood fluid down an irregular conduit. The fluid exhibits electrical conductivity, and an external uniform magnetic field is added to this motion. Heat and mass transmission are considered, leading to an examination of the Dufour and Soret effects. Al-Khafajy and Mohammed[25] examined a mathematical model elucidating the effects of thermal transfer on the oscillating flow of Bingham fluid with changing viscosity in a porous channel within the context of magnetohydrodynamics. Salahuddin et al.[26] performed a study analyzing numerical behavior utilizing the Adams-Bashforth predictor-corrector method of numerical analysis for the Williamson fluid model, considering variable viscosity, natural convection, and an angled magnetic field. Additionally, thermal radiation, Joule heating, and heat source/sink effects are incorporated into the thermal considerations. Akram et al.[27] studied the peristaltic flow of Bingham fluids under an inclined magnetic field, while Humnekar and Darbhasayanam[28] investigated variable-viscosity nanofluid flow in inclined porous media. These recent studies highlight the ongoing need for improved formulations that incorporate thermal, magnetic, and viscous variations simultaneously. Recent works have also extended non-Newtonian flow analysis by incorporating advanced thermal and concentration mechanisms. Panda et al.[29] examined Marangoni convection in a tangent hyperbolic fluid with Cattaneo–Christov heat and mass flux, buoyancy, and activation energy effects. Sneha et al.[30] considered MHD Marangoni nano-boundary-layer flow with carbon nanotubes and thermal radiation, highlighting the influence of magnetic and radiative effects on heat transfer. More closely related to yield-stress rheology, Sachhin et al.[31] analyzed thermodynamic and viscosity-ratio effects in a non-Newtonian tetra-Bingham nanofluid flow, showing the importance of rheological parameters in controlling transport behavior.

In an inclined channel, the influence of gravity is crucial in propelling the flow. The gravitational force acting along the slope of the channel affects the overcoming of yield stress, allowing the fluid to flow. In the case of Bingham fluids, flow takes place when the shear stress surpasses the yield stress. The angle of inclination in a channel influences the critical shear stress required to initiate flow. Lakshminarayana et al.[32] investigated the concurrent impacts of heated Joules and slip on the peristaltic flow of a Bingham fluid within an inclination tapered permeable channel featuring elastic walls. Mohammed and Al-Khafajy[33] examined the impact of temperature and concentration on magnetohydrodynamic oscillatory flow of Bingham fluid with varying viscosity in a sloped channel. umbinarasaiah[34] conducted a numerical investigation of entropy generation in an incompressible Casson fluid moving through an inclined permeable channel subjected to magnetic influence. Jha and Aina[35] offered a mathematical model for the complete magnetohydrodynamic mixed convection flow of an electrically conducting, viscous, incompressible fluid in an inclined permeable channel subject to time-periodic boundary constraints.

Although the above studies provide useful insights into oscillatory MHD convection, porous-medium transport, non-Newtonian rheology, and coupled heat and mass transfer, most of them address these mechanisms separately or in different geometrical configurations. Analytical studies combining oscillatory MHD mixed convection, Bingham yield-stress behavior, variable viscosity, porous-medium resistance, inclination effects, and coupled thermal and concentration transport in a single channel-flow formulation remain limited. Therefore, the novelty of the present work lies in developing an analytical model that brings these interacting mechanisms together and clarifies how the competition between buoyancy forces, Lorentz-force damping, yield-stress resistance, viscosity variation, and porous drag controls the velocity, temperature, and concentration fields.

This study comprises five sections. The initial section is the introduction, which encompasses a historical review of the factors being examined. The second involves constructing the mathematical model. The third portion presents the resolution to the issue. The fourth portion encompasses an analysis of the outcomes via the function diagrams we acquired. The investigation was ended with a discussion and conclusions.

2. Mathematical formulation

Let us consider the flow of a non-Newtonian (Bingham) fluid with variable viscosity under the effects of radioactive heat transfer and electrically-applied magnetic field as depicted through an inclined porous channel with a width of h (Figure 1). Fluids are supposed to have minimal electromagnetic power produced with a low electrical conductivity. We think of the system of Cartesian coordinates so that is the velocity vector.

The basic equations governing the problem are provided as:

The continuity equation is given by:

$$\nabla.\mathcal{U}=0 \tag{1}$$

The momentum equations:

$$\rho\left(\overline{\mathcal{U}}.\nabla\right)\overline{\mathcal{U}}=\nabla.\overline{\mathbb{S}}+\overline{J}\times\overline{B}-\frac{\mu\left(\overline{\mathcal{T}}\right)}{k}\overline{\mathcal{U}}+\rho g\mathcal{B}_{\mathcal{T}}\sin\varphi\Delta\overline{\mathcal{T}}+\rho g\mathcal{B}_{\mathcal{C}}\sin\varphi\Delta\overline{\mathcal{C}}+\rho g(isin(\phi)-jcos(\phi)) \tag{2}$$

The concentration equation:

$$\left(\overline{\mathcal{U}}.\nabla\right)\overline{\mathcal{C}}=\nabla.\left(\frac{D_{\mathcal{C}}\mathcal{T}_d}{\mathcal{T}_m}\nabla\overline{\mathcal{T}}+D_{\mathcal{C}}\nabla\overline{\mathcal{C}}\right)-K_r^*\Delta\overline{\mathcal{C}} \tag{3}$$

The temperature equation:

$$\rho c_{\mathcal{T}}\left(\overline{\mathcal{U}}.\nabla\right)\overline{\mathcal{T}}=\nabla.\left(K_{\mathcal{T}}\nabla\overline{\mathcal{T}}\right)+\Delta.Q_{\mathcal{T}}+Q_H\Delta\overline{\mathcal{T}} \tag{4}$$

where $\overline{\mathcal{U}}=(\overline{u}(\overline{y},\overline{t}),0,0)$ is the velocity field, $\overline{\mathcal{T}}=\overline{\mathcal{T}}(\overline{y},\overline{t})$ "temperature", $\overline{\mathcal{C}}=\overline{\mathcal{C}}(\overline{y},\overline{t})$ "concentration", $\overline{J}\times\overline{B}=-\sigma B_s^2 sin^2\varphi\, \mathcal{U}i$ "Lorentz force for inclined magnetic field strength",[33] σ is a conductivity of the fluid, k is a permeability, ρ "fluid density", g "gravity field", $c_{\mathcal{T}}$ "specific heat at constant pressure", $K_{\mathcal{T}}$ "thermal conductivity", Q_H "heat generation", $D_{\mathcal{C}}$ "coefficient of mass diffusivity" and $\mathcal{T}_d$ "thermal diffusion ratio", $\Delta.Q_{\mathcal{T}}=4\alpha^2\left(\overline{\mathcal{T}}-\mathcal{T}_0\right)$"radiation heat flux",[36] α "radiation absorption", $(0\le\phi\le\pi)$ is the angle between the centre channel and the ground acceleration. The corresponding boundary conditions are given below:

$$\overline{u}=0,\overline{\mathcal{T}}=\mathcal{T}_0,\overline{\mathcal{C}}=\mathcal{C}_h \text{ at } \overline{y}=0 \text{ and } \overline{u}=0,\overline{\mathcal{T}}=\mathcal{T}_h,\overline{\mathcal{C}}=\mathcal{C}_0 \text{ at } \overline{y}=h \tag{5}$$

The basic equation for the Bingham fluid,[32] given by:

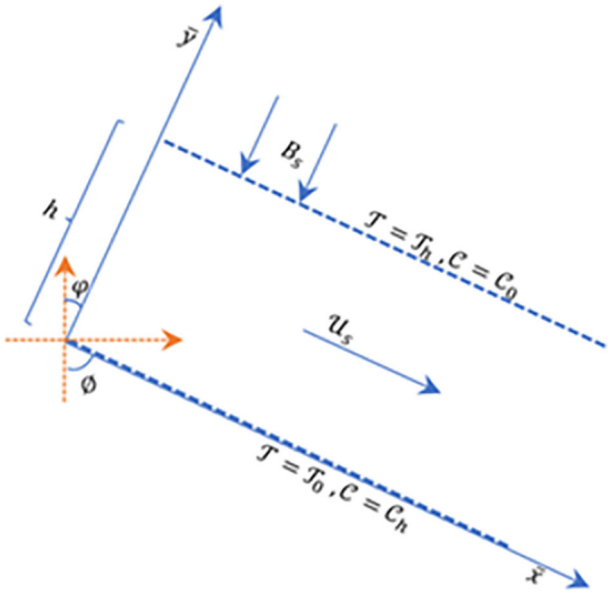


Figure 1. Physical model of the inclined porous channel.

$$\overline{\mathbb{S}} = -\overline{p}\boldsymbol{I} + \overline{\mathbb{s}} \text{ and } \overline{\mathbb{s}} = \begin{cases} \left(\mu\left(\overline{\mathcal{T}}\right) + \dfrac{\tau_0}{\left(\dfrac{\partial \overline{u}}{\partial \overline{y}}\right)}\right)\left(\nabla\overline{\mathcal{U}} + \left(\nabla\overline{\mathcal{U}}\right)^T\right) & \text{for} \quad \tau \geq \tau_0 \\ 0 & \text{for} \quad \tau < \tau_0 \end{cases} \tag{6}$$

Where $\overline{p}$ "pressure", $\boldsymbol{I}$ "unit tensor", μ_c "fluid viscosity", τ_0 "yield stress", and $\overline{\dot{\gamma}}$ "shear rate".

When compensating for the velocity vector and the concentration and temperature functions, taking into account the magnetic field generated by the passage of a simple electric current over a porous wall of the inclined flow channel, and by compensating for the shear stress (Equation 6), and after simplifying, we rewrite the nonlinear partial differential system (1)-(4) as follows:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{7}$$

$$\rho\left(\frac{\partial \overline{u}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}}\right) = -\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{\partial \overline{\mathbb{s}}_{11}}{\partial \overline{x}} + \frac{\partial \overline{\mathbb{s}}_{12}}{\partial \overline{y}} - \sigma B_s^2 \sin^2\varphi\, \overline{u} - \frac{\mu\left(\overline{\mathcal{T}}\right)}{k}\overline{u} + \rho g \mathcal{B}_{\mathcal{T}} \sin\varphi\left(\overline{\mathcal{T}} - \mathcal{T}_0\right)$$
$$+\rho g \mathcal{B}_{\mathcal{C}} \sin\varphi\left(\overline{\mathcal{C}} - \mathcal{C}_0\right) + \rho g \sin\phi \tag{8}$$

$$\rho\left(\frac{\partial \overline{v}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{v}}{\partial \overline{y}}\right) = -\frac{\partial \overline{p}}{\partial \overline{y}} + \frac{\partial \overline{\mathbb{s}}_{21}}{\partial \overline{x}} + \frac{\partial \overline{\mathbb{s}}_{22}}{\partial \overline{y}} - \frac{\mu\left(\overline{\mathcal{T}}\right)}{k}\overline{v} \tag{9}$$

$$\frac{\partial \overline{\mathcal{C}}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{\mathcal{C}}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{\mathcal{C}}}{\partial \overline{y}} = \frac{D_{\mathcal{C}}\mathcal{T}_d}{\mathcal{T}_m}\left(\frac{\partial^2 \overline{\mathcal{T}}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\mathcal{T}}}{\partial \overline{y}^2}\right) + D_{\mathcal{C}}\left(\frac{\partial^2 \overline{\mathcal{C}}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\mathcal{C}}}{\partial \overline{y}^2}\right) - K_r^*\left(\overline{\mathcal{C}} - \mathcal{C}_0\right) \tag{10}$$

$$c_{\mathcal{T}}\rho\left(\frac{\partial \overline{\mathcal{T}}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{\mathcal{T}}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{\mathcal{T}}}{\partial \overline{y}}\right) = K_{\mathcal{T}}\left(\frac{\partial^2 \overline{\mathcal{T}}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\mathcal{T}}}{\partial \overline{y}^2}\right) - 4\alpha^2\left(\mathcal{T}_0 - \overline{\mathcal{T}}\right) + Q_H\left(\overline{\mathcal{T}} - \mathcal{T}_0\right) \tag{11}$$

The stress components are given by:

$$\overline{\mathbb{s}}_{11} = \overline{\mathbb{s}}_{22} = 0 \text{ and } \overline{\mathbb{s}}_{12} = \overline{\mathbb{s}}_{21} = \left(\mu\left(\overline{\mathcal{T}}\right) + \frac{\tau_0}{\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)}\right)\frac{\partial \overline{u}}{\partial \overline{y}} = \mu\left(\overline{\mathcal{T}}\right)\frac{\partial \overline{u}}{\partial \overline{y}} + \tau_0 \tag{12}$$

By substituting the equation (12) into the governing equations and after simplifying, we obtain:

$$\rho\frac{\partial \overline{u}}{\partial \overline{t}} = -\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{\partial}{\partial \overline{y}}\left(\mu\left(\overline{\mathcal{T}}\right)\frac{\partial \overline{u}}{\partial \overline{y}} + \tau_0\right) - \sigma B_s^2 \sin^2\varphi\, \overline{u} - \frac{\mu\left(\overline{\mathcal{T}}\right)}{k}\overline{u} + \rho g \sin\varphi\left(\mathcal{B}_{\mathcal{T}}\left(\overline{\mathcal{T}} - \mathcal{T}_0\right) + \mathcal{B}_{\mathcal{C}}\left(\overline{\mathcal{C}} - \mathcal{C}_0\right)\right) + \rho g \sin\phi \tag{13}$$

$$\frac{\partial \overline{p}}{\partial \overline{y}} = 0 \tag{14}$$

$$\frac{\partial \overline{\mathcal{C}}}{\partial \overline{t}} = \frac{D_{\mathcal{C}}\mathcal{T}_d}{\mathcal{T}_m}\frac{\partial^2 \overline{\mathcal{T}}}{\partial \overline{y}^2} + D_{\mathcal{C}}\frac{\partial^2 \overline{\mathcal{C}}}{\partial \overline{y}^2} - K_r^*\left(\overline{\mathcal{C}} - \mathcal{C}_0\right) \tag{15}$$

$$c_{\mathcal{T}}\rho\frac{\partial \overline{\mathcal{T}}}{\partial \overline{t}} = K_{\mathcal{T}}\frac{\partial^2 \overline{\mathcal{T}}}{\partial \overline{y}^2} + \left(4\alpha^2 + Q_H\right)\left(\overline{\mathcal{T}} - \mathcal{T}_0\right) \tag{16}$$

3. Method of solution

The equation (14) shows that the pressure does not depend on y. To solve the above system of equations, we use its non-dimensional conditions as follows:

By substituting equations from Table 1 into equations (13), (15), (16) and the equations of boundary conditions equation (5), we have the following non-dimensional equations:

Table 1. Dimensionless parameters and their physical significance.

Symbol	Parameter name	Definition/ Expression	Physical interpretation
Re	Reynolds number	$Re = \frac{\rho h \mathcal{U}_s}{\mu_e}$	Ratio of inertial to viscous forces; characterizes flow regime.
Pe	Péclet number	$Pe = \frac{\rho h \mathcal{U}_s c_{\mathcal{T}}}{K_T}$	Measures the relative importance of convective to conductive heat transfer.
Sc	Schmidt number	$Sc = \frac{\mu_e}{\rho D_{\mathcal{C}}}$	Ratio of momentum diffusivity to mass diffusivity; governs concentration boundary layer thickness.
M	Magnetic parameter (Hartmann number squared)	$M = \sqrt{\frac{\sigma B_0^2 h^2}{\mu_e}}$	Represents the relative influence of Lorentz (magnetic) forces over viscous forces.
Da	Darcy number	$Da = \frac{k}{h^2}$	Indicates the permeability effect of the porous medium on the flow.
F_r	Froude number	$F_r = \frac{\mathcal{U}_s^2}{gh}$	Expresses the ratio of inertial forces to gravitational forces.
B_n	Bingham number	$B_n = \tau_0 \frac{h}{\mu_e \mathcal{U}_s}$	Quantifies yield stress effects; higher values indicate stronger resistance before flow begins.
$G_{\mathcal{T}}$	Thermal Grashof number	$G_{\mathcal{T}} = \frac{\rho g \mathcal{B}_{\mathcal{T}} h^2 (\mathcal{T}_h - \mathcal{T}_0)}{\mu_e \mathcal{U}_s}$	Represents buoyancy forces due to temperature differences.
$G_{\mathcal{C}}$	Solutal Grashof number	$G_{\mathcal{C}} = \frac{\rho g \mathcal{B}_{\mathcal{C}} h^2 (\mathcal{C}_h - \mathcal{C}_0)}{\mu_e \mathcal{U}_s}$	Represents buoyancy effects due to concentration differences.
$\mathcal{K}_{\mathcal{T}}$	Radiation parameter	$\mathcal{K}_{\mathcal{T}} = \frac{4a^2 h^2}{K_{\mathcal{T}}}$	Measures the contribution of radiative heat transfer relative to conduction.
$\mathcal{Q}_H$	Heat generation parameter	$\mathcal{Q}_H = \frac{Q_H h^2}{K_{\mathcal{T}}}$	Quantifies internal heat generation or absorption within the fluid.
$S_{\mathcal{T}}$	Soret number	$S_{\mathcal{T}} = \frac{D_{\mathcal{C}} \mathcal{T}_d (\mathcal{T}_h - \mathcal{T}_0)}{h \mathcal{T}_m \mathcal{U}_s (\mathcal{C}_h - \mathcal{C}_0)}$	Captures thermodiffusion effects (mass flux due to temperature gradients).
$\mathcal{K}_{\mathcal{C}}$	Chemical reaction parameter	$\mathcal{K}_{\mathcal{C}} = \frac{hK_c^*}{\mathcal{U}_s}$	Represents the rate of chemical reaction relative to convective transport.
ϕ	Channel inclination angle	-	Angle between channel axis and horizontal plane; controls gravity component along the flow.
φ	Magnetic field inclination angle	-	Angle between magnetic field and vertical direction; controls Lorentz force orientation.

$$R_e \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu(\mathcal{T}) \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial \mu(\mathcal{T})}{\partial y} - \left(M^2 sin^2 \varphi + \frac{\mu(\mathcal{T})}{Da} \right) u + \sin\varphi (G_{\mathcal{T}} \mathcal{T} + G_{\mathcal{C}} \mathcal{C}) + \frac{R_e}{F_r} \sin\phi \tag{17}$$

$$\frac{\partial \mathcal{C}}{\partial t} = S_{\mathcal{T}} \frac{\partial^2 \mathcal{T}}{\partial y^2} + \frac{1}{S_{\mathcal{C}}} \frac{\partial^2 \mathcal{C}}{\partial y^2} - \mathcal{K}_{\mathcal{C}} \mathcal{C} \tag{18}$$

$$Pe \frac{\partial \mathcal{T}}{\partial t} = \frac{\partial^2 \mathcal{T}}{\partial y^2} + (\mathcal{K}_{\mathcal{T}} + \mathcal{Q}_H) \mathcal{T} \tag{19}$$

$$u = 0, \mathcal{T} = 0, \mathcal{C} = 1 \text{ at } y = 0 \text{ and } u = 0, \mathcal{T} = 1, \mathcal{C} = 0 \text{ at } y = 1 \tag{20}$$

3.1 Solution of the problem

This section includes solving a system of differential equations. We begin by solving the heat equation, passing through the concentration equation, and then we end by solving the velocity equation.

3.1.1 Solution of the heat and concentration equations

Using the separating variables method to solve the heat equation (19) and the concentration equation (18) with the boundary conditions equation (20), respectively. Let ω be the frequency of oscillation, and let

$$\mathcal{T}(y,t) = \mathcal{T}_0(y)\, e^{i\omega t} \text{ and } \mathcal{C}(y,t) = \mathcal{C}_0(y) e^{i\omega t} \tag{21}$$

By substituting equation (21) into equations (19) and (18), respectively, and after simplifying the two equations, we obtain

$$\frac{\partial^2 \mathcal{C}_0(y)}{\partial y^2} + S_e S_{\mathcal{T}} \frac{\partial^2 \mathcal{T}_0(y)}{\partial y^2} - (S_e \mathcal{K}_e + i\omega S_e)\, \mathcal{C}_0(y) = 0$$

$$\frac{\partial^2 \mathcal{T}_0(y)}{\partial y^2} + (\mathcal{K}_{\mathcal{T}} + \mathcal{Q}_H - Pei\omega)\mathcal{T}_0(y) = 0$$

with boundary conditions $\mathcal{T}_0(0) = \mathcal{C}_0(1) = 0$ and $\mathcal{T}_0(1) = \mathcal{C}_0(0) = e^{-i\omega t}$.

The solution of temperature equation is

$$\mathcal{T}(y,t) = \text{Csc}\left[\sqrt{H}\right] \text{Sin}\left[\sqrt{H} y\right]$$

The solution of concentration equation is

$$\begin{aligned}\mathcal{C}(y,t) = e^{it\omega} \Bigg(& \left(-\frac{e^{-it\omega + \sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}} - \frac{e^{-it\omega} H S_e S_{\mathcal{T}}}{H + i\omega S_e + \mathcal{K}_e S_e}}{e^{-\sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}} - e^{\sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}}} \right) e^{-y\sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}} \\ & + \left(\frac{e^{-it\omega}\left(-H - i\omega S_e - \mathcal{K}_e S_e + e^{\sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}} H S_e S_{\mathcal{T}} \right)}{\left(-1 + e^{2\sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}}\right)(H + i\omega S_e + \mathcal{K}_e S_e)} \right) e^{y\sqrt{i\omega + \mathcal{K}_e}\sqrt{S_e}} \\ & - \frac{e^{-it\omega} H \text{Csc}\left[\sqrt{H}\right] \text{Sin}\left[\sqrt{H} y\right] S_e S_{\mathcal{T}}}{H + i\omega S_e + \mathcal{K}_e S_e} \Bigg)\end{aligned}$$

where $H = \mathcal{K}_{\mathcal{T}} + \mathcal{Q}_H - Pei\omega$.

3.1.2 Solution of the momentum equations

The Reynolds model for the variation of viscosity with temperature is $\mu(\mathcal{T}) = \exp(-\epsilon\mathcal{T})$, taking the Maclaurin's expansion, we get $\mu(\mathcal{T}) = 1 - \epsilon\mathcal{T}$, $\epsilon \ll 1$.

Using the separating variables method to solve the momentum equation (17) with the boundary conditions equation (20). Let

$$u(y,t) = u_0(y) e^{i\omega t} \text{ and } \frac{dp}{dx} = -\lambda e^{i\omega t} \tag{22}$$

Where λ is a real pressure constant. Substituting Maclaurin's formula for the fluid viscosity variable in addition to the (22) equation in the (17) equation, we get

$$\begin{aligned} R_e i\omega\, u_0(y) = {} & \lambda + (1 - \epsilon\mathcal{T}) \frac{\partial^2 (u_0(y))}{\partial y^2} - \epsilon \frac{\partial (u_0(y))}{\partial y} \frac{\partial \mathcal{T}}{\partial y} - \left(M^2 sin^2\varphi + \frac{1}{Da} \right) u_0(y) + \frac{\epsilon\mathcal{T}}{Da} u_0(y) + \sin\varphi (G_{\mathcal{T}} \mathcal{T}_0 + G_{\mathcal{C}} \mathcal{C}_0) \\ & + e^{-i\omega t} \frac{R_e}{F_r} \sin\phi \end{aligned} \tag{23}$$

The equation's solution will be examined in two distinct cases.

Case I (when $\epsilon = 0$)

In the specific instance when $\epsilon = 0$, indicating that the viscosity remains constant, we obtain from (23)

$$\frac{\partial^2 u_0(y)}{\partial y^2} - \left(M^2 sin^2\varphi + \frac{1}{Da} + i\omega R_e\right) u_0(y) = -\left(\lambda + e^{-i\omega t}\left(G_{\mathcal{T}} \sin\varphi \mathcal{T} + G_{\mathcal{C}} \sin\varphi \mathcal{C} + \frac{R_e}{F_r}\sin\phi\right)\right) \tag{24}$$

with boundary conditions: $u_0(0) = u_0(1) = 0$.

Due to the complexity of solving the velocity equation, we shall analyze the behavior of the solution by graphing the function rather than deriving its formula.

Case II (when $\epsilon \neq 0$)

Equation (23) is a nonlinear differential equation and it is hard to find an exact solution, so we will use the perturbation technique to find the solution to the problem as follows:

$$u_0(y) = u_{00}(y) + \epsilon u_{01}(y) + o(\epsilon^2) \tag{25}$$

Substituting equation (25) in equation (23), we obtain:

$$\begin{aligned} R_e i\omega\left(u_{00}(y) + \epsilon u_{01}(y)\right) = \lambda + (1 - \epsilon\mathcal{T})\frac{\partial^2}{\partial y^2}\left(u_{00}(y) + \epsilon u_{01}(y)\right) - \epsilon\frac{\partial \mathcal{T}}{\partial y}\frac{\partial}{\partial y}\left(u_{00}(y) + \epsilon u_{01}(y)\right) \\ - \left(M^2 sin^2\varphi + \frac{1}{Da}\right)\left(u_{00}(y) + \epsilon u_{01}(y)\right) + \frac{\epsilon\mathcal{T}}{Da}\left(u_{00}(y) + \epsilon u_{01}(y)\right) + \sin\varphi G_{\mathcal{T}}\mathcal{T}_0 \\ + \sin\varphi G_{\mathcal{C}}\mathcal{C}_0 + e^{-i\omega t}\frac{R_e}{F_r}\sin\phi \end{aligned}$$

Equating the like powers of ϵ, we obtain the following results presented in the forthcoming subsections:

i. **Zero-order system**

$$\frac{\partial^2 u_{00}(y)}{\partial y^2} - \left(M^2 sin^2\varphi + \frac{1}{Da} + R_e i\omega\right) u_{00}(y) = -\left(\lambda + G_{\mathcal{T}} \sin\varphi \mathcal{T}_0 + \sin\varphi G_{\mathcal{C}} \mathcal{C}_0 + e^{-i\omega t}\frac{R_e}{F_r}\sin\phi\right) \tag{26}$$

It is consistent with the Case I.

ii. **First-order system**

$$\frac{\partial^2 u_{01}(y)}{\partial y^2} - \left(M^2 sin^2\varphi + \frac{1}{Da} + R_e i\omega\right) u_{01}(y) = \mathcal{T}\left(\frac{\partial^2 u_{00}(y)}{\partial y^2} - \frac{1}{Da}u_{00}(y)\right) + \frac{\partial \mathcal{T}}{\partial y}\frac{\partial u_{00}(y)}{\partial y} \tag{27}$$

with the boundary conditions: $u_{01}(0) = u_{01}(1) = 0$

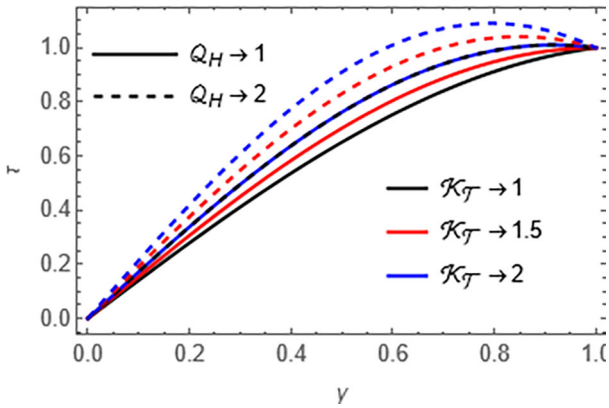


Figure 2. Temperature graph for values $\mathcal{K}_{\mathcal{T}}$ and $\mathcal{Q}_H$ with $\omega = 1, Pe = 0.7$.

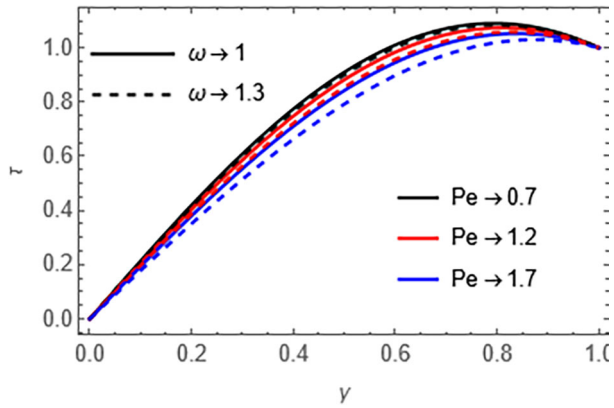


Figure 3. **Temperature graph for values Pe and ω with $\mathcal{K}_{\mathcal{T}} = 2, \mathcal{Q}_H = 2$.**

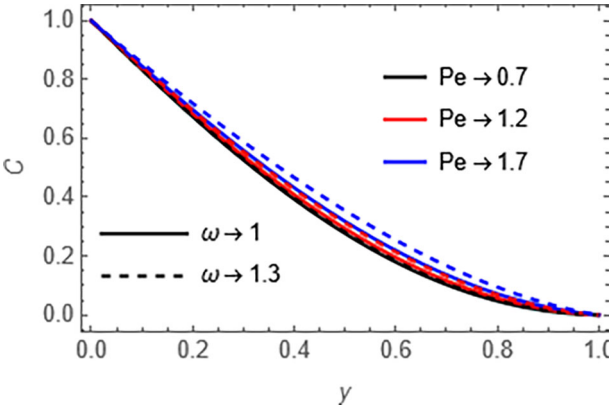


Figure 4. **Concentration graph for values Pe and ω with $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, S_{\mathcal{C}} = S_{\mathcal{T}} = \mathcal{K}_{\mathcal{C}} = 0.7$.**

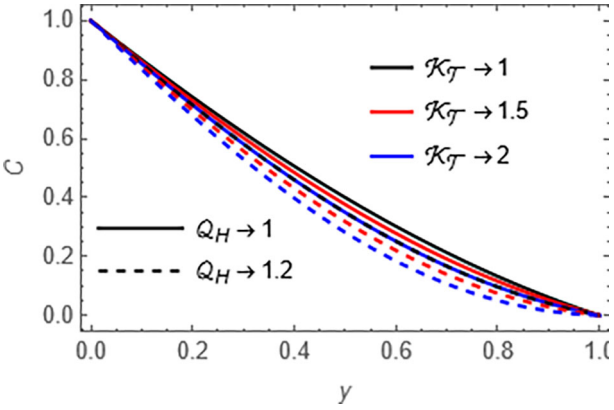


Figure 5. **Concentration graph for values $\mathcal{K}_{\mathcal{T}}$ and $\mathcal{Q}_H$ with $\omega = 1, Pe = S_{\mathcal{C}} = S_{\mathcal{T}} = \mathcal{K}_{\mathcal{C}} = 0.7$.**

The nonlinear characteristics of equation (27) render the derivation of an exact analytical formula for velocity difficult. Consequently, we employed an approximate method utilizing perturbation and separation of variables to examine the effects of varying viscosity and other flow characteristics. In the current investigation, the governing equations initially excluded the explicit description of the yield-stress factor related to Bingham rheology. To overcome this constraint and

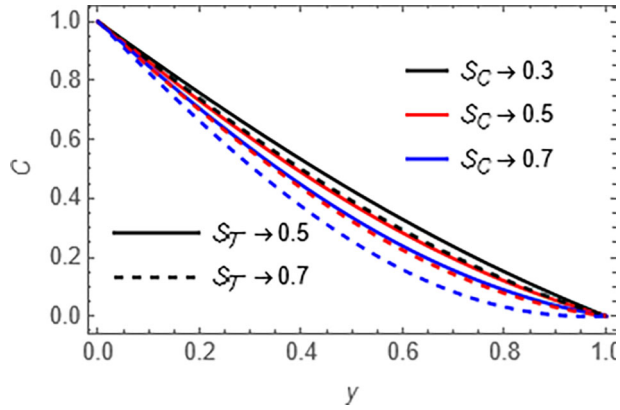


Figure 6. **Concentration graph for values $S_{\mathcal{C}}$ and $S_{\mathcal{T}}$ with $\mathcal{K}_{\mathcal{T}} = Q_H = 2, \omega = 1, Pe = \mathcal{K}_{\mathcal{C}} = 0.7$.**

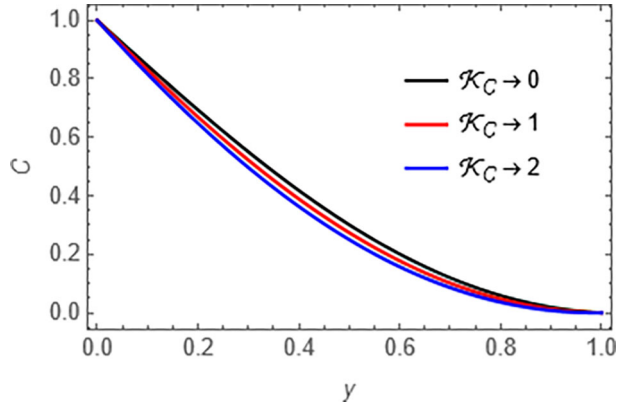


Figure 7. **Concentration graph for values $\mathcal{K}_{\mathcal{C}}$with $\mathcal{K}_{\mathcal{T}} = Q_H = 2, \omega = 1, Pe = S_{\mathcal{C}} = S_{\mathcal{T}} = 0.7$.**

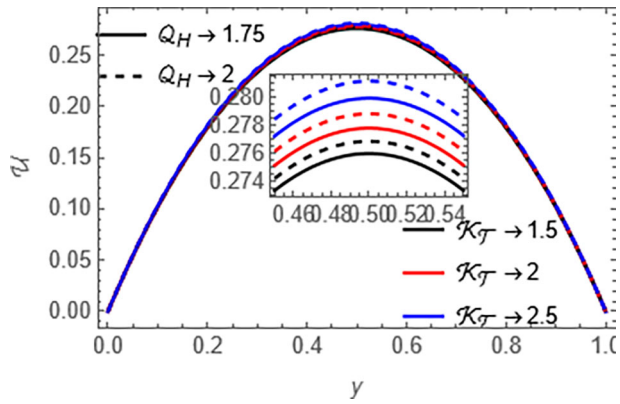


Figure 8. **Velocity graph for values $Q_H$and $\mathcal{K}_{\mathcal{T}}$ with $\omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0$.**

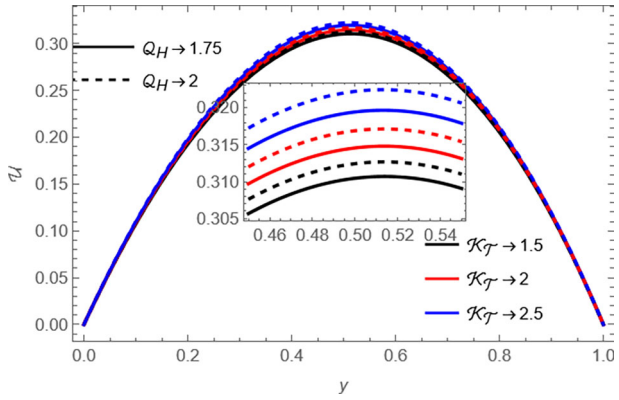


Figure 9. **Velocity graph for values** $\mathcal{Q}_H$**and** $\mathcal{K}_\mathcal{T}$ **with** $\omega = 1, \lambda = 0.6, S_\mathcal{T} = 0.3, Pe = S_\mathcal{C} = \mathcal{K}_\mathcal{C} = Da = 0.7, M = 1.1, G_\mathcal{T} = G_\mathcal{C} = 0.5, F_r = 0.8,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.2.$

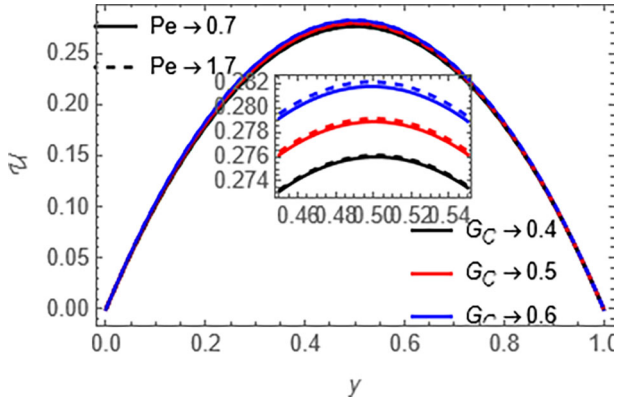


Figure 10. **Velocity graph for values** Pe**and** $G_\mathcal{C}$ **with** $\mathcal{K}_\mathcal{T} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_\mathcal{T} = 0.3, S_\mathcal{C} = \mathcal{K}_\mathcal{C} = Da = 0.7, M = 1.1, G_\mathcal{T} = 0.5, F_r = 0.8,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.$

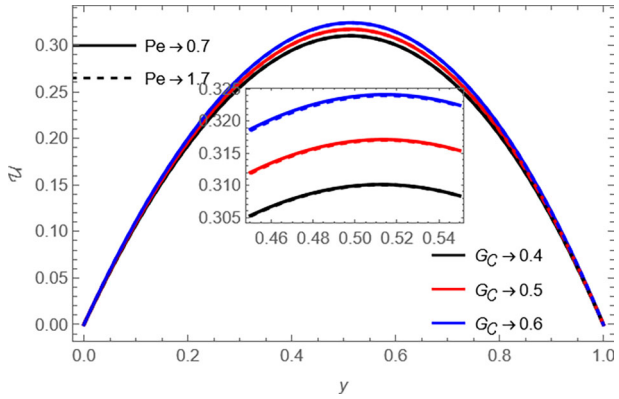


Figure 11. **Velocity graph for values** Pe**and** $G_\mathcal{C}$ **with** $\mathcal{K}_\mathcal{T} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_\mathcal{T} = 0.3, S_\mathcal{C} = \mathcal{K}_\mathcal{C} = Da = 0.7, M = 1.1, G_\mathcal{T} = 0.5, F_r = 0.8,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.2.$

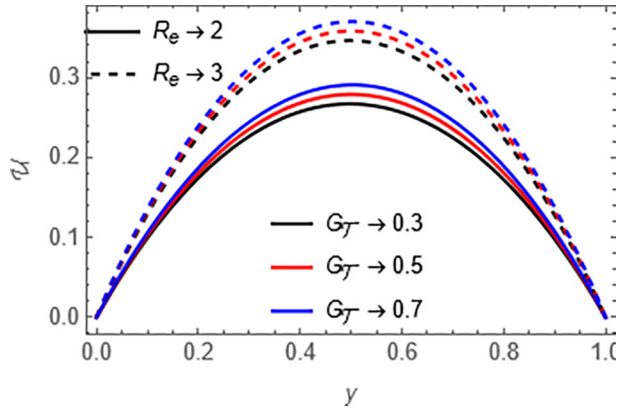


Figure 12. **Velocity graph for values** R_e **and** $G_{\mathcal{T}}$ **with** $\mathcal{K}_{\mathcal{T}} = \Omega_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{C}} = 0.5, F_r = 0.8, \varphi = \phi = \pi/4, \epsilon = 0.$

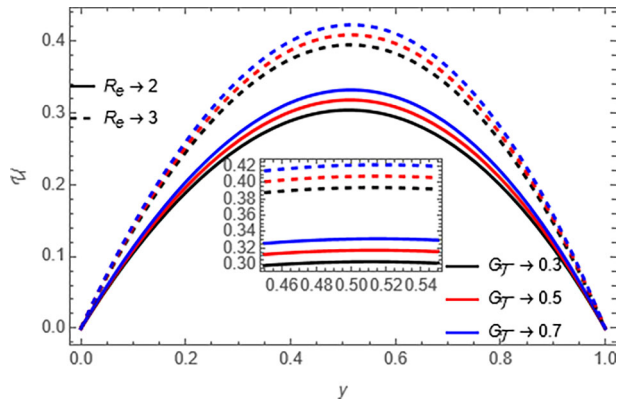


Figure 13. **Velocity graph for values** R_e **and** $G_{\mathcal{T}}$ **with** $\mathcal{K}_{\mathcal{T}} = \Omega_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{C}} = 0.5, F_r = 0.8, \varphi = \phi = \pi/4, \epsilon = 0.2.$

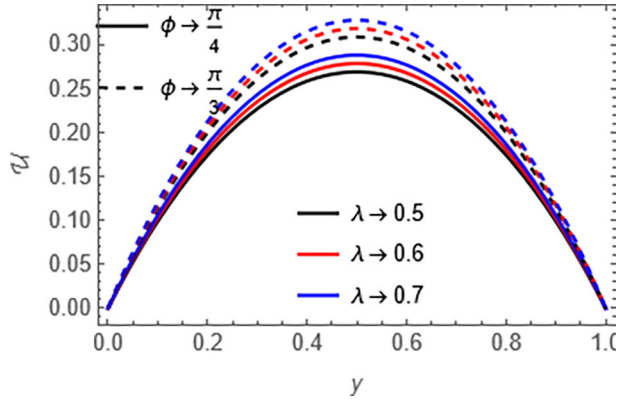


Figure 14. **Velocity graph for values** λ **and** ϕ **with** $\mathcal{K}_{\mathcal{T}} = \Omega_H = 2, \omega = 1, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \varphi = \pi/4, \epsilon = 0.$

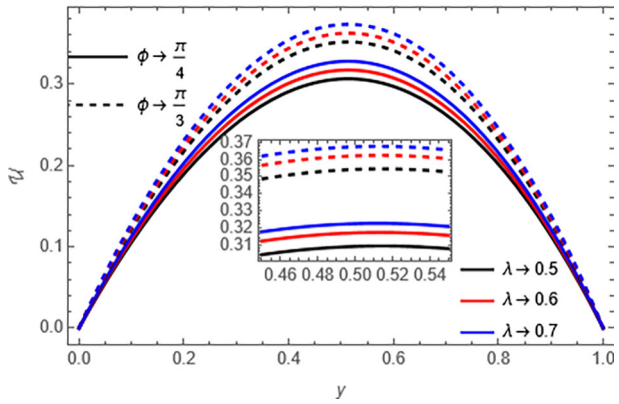


Figure 15. **Velocity graph for values λ and ϕ with $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \varphi = \pi/4, \epsilon = 0.2.$**

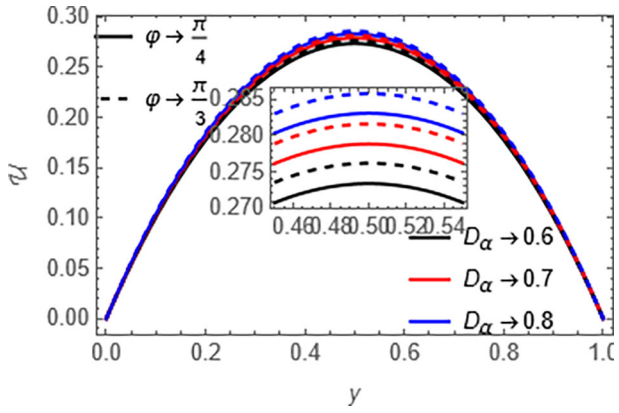


Figure 16. **Velocity graph for values Da and φ with $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \phi = \pi/4, \epsilon = 0.$**

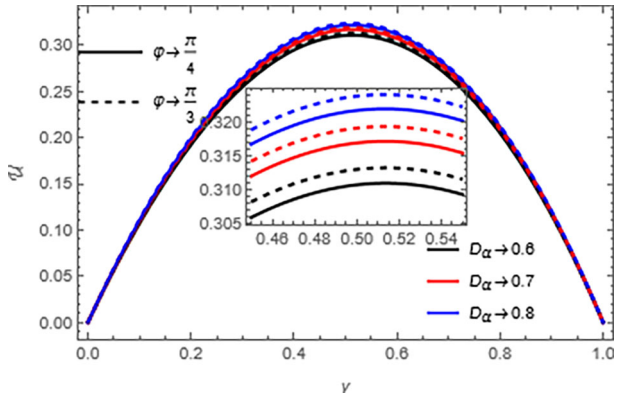


Figure 17. **Velocity graph for values Da and φ with $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \phi = \pi/4, \epsilon = 0.2.$**

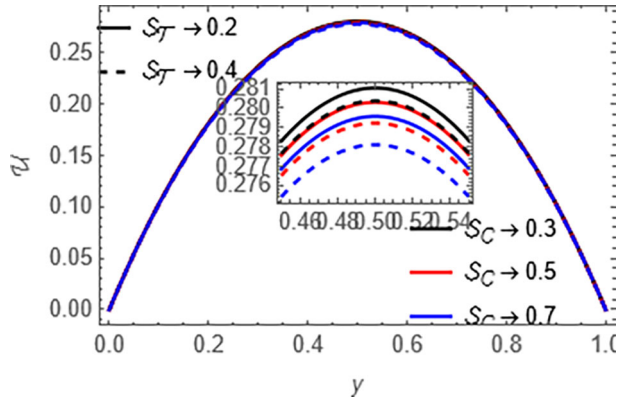


Figure 18. **Velocity graph for values** $S_{\mathcal{C}}$ **and** $S_{\mathcal{T}}$ **with** $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, Pe = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.$

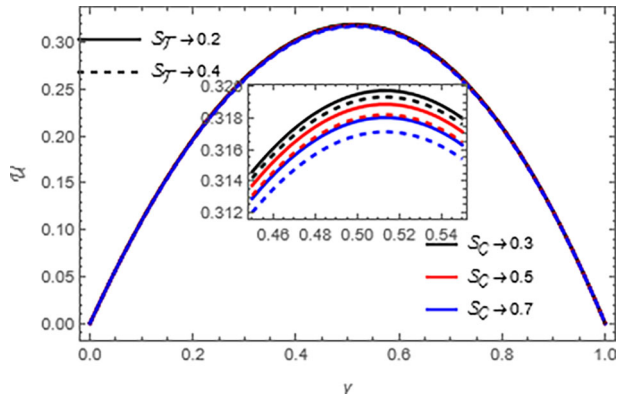


Figure 19. **Velocity graph for values** $S_{\mathcal{C}}$ **and** $S_{\mathcal{T}}$ **with** $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, Pe = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8, R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.2.$

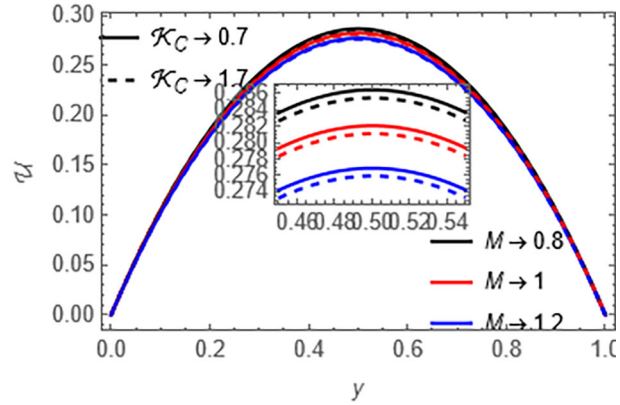


Figure 20. **Velocity graph for values** $\mathcal{K}_{\mathcal{C}}$ **and** M **with** $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3,\ Pe = S_{\mathcal{C}} = Da = 0.7, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.$

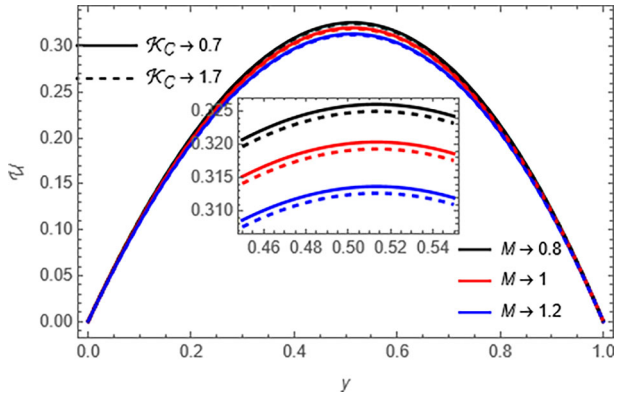


Figure 21. **Velocity graph for values** $\mathcal{K}_{\mathcal{C}}$ **and** M **with** $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3,\ Pe = S_{\mathcal{C}} = Da = 0.7, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.2.$

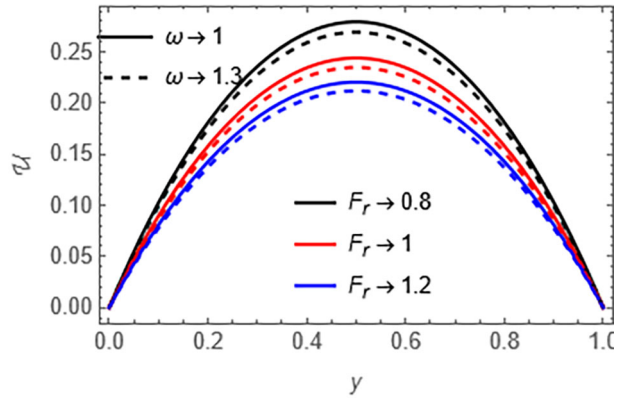


Figure 22. **Velocity graph for values** F_r **and** ω **with** $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.$

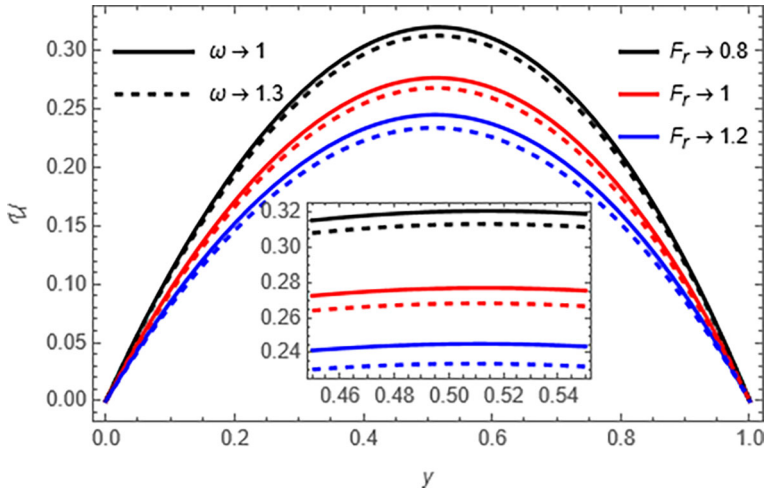


Figure 23. **Velocity graph for values** F_r **and** ω **with** $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \lambda = 0.6, S_{\mathcal{T}} = 0.3, Pe = S_{\mathcal{C}} = \mathcal{K}_{\mathcal{C}} = Da = 0.7, M = 1.1, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5,\ R_e = 2, \varphi = \phi = \pi/4, \epsilon = 0.2.$

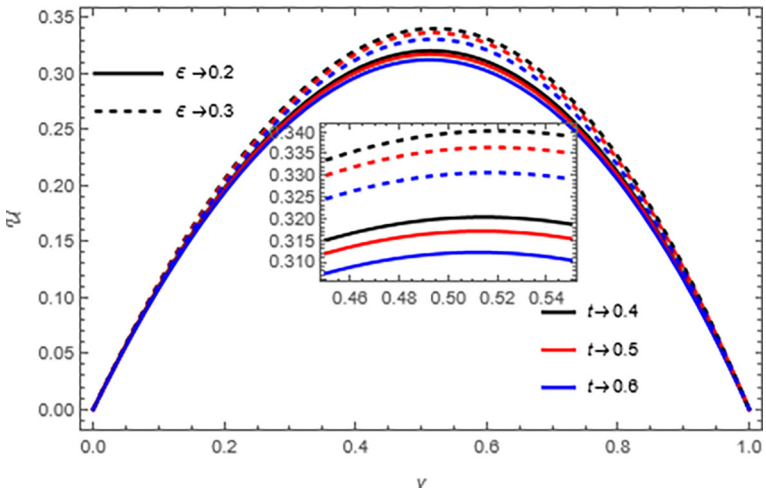


Figure 24. Velocity graph for values t and ϵ with $\mathcal{K}_{\mathcal{T}} = \mathcal{Q}_H = 2, \omega = 1, \lambda = 0.6, S_{\mathcal{T}} = 0.3,$ $Pe = \mathcal{K}_{\mathcal{C}} = S_{\mathcal{C}} = Da = 0.7, G_{\mathcal{T}} = G_{\mathcal{C}} = 0.5, F_r = 0.8,$ $R_e = 2, \varphi = \phi = \pi/4, M = 1.1.$

offer a more physically accurate depiction of the flow, the formulation has been enhanced to explicitly include the yield criterion and the Bingham number (B), which denotes the ratio of yield to viscous stresses. The updated formulation is detailed in the subsequent subsection.

3.2 Bingham regularization and yield surface

The momentum equation was revised to explicitly include the yield-stress impact through the Papanastasiou regularization [Papanastasiou, 1987]. This formulation eliminates discontinuities between yielded and unyielded regions, facilitating seamless numerical analysis. The effective shear stress is written as:

$$\tau = \tau_0[1 - \exp(-m\dot{\gamma})] + \mu c\dot{\gamma},$$

where τ_0 represents the yield stress. Shear rate is denoted as $\dot{\gamma}$, and m represents a significant regularization parameter that governs the abruptness of the transition between the unyielded and yielded states. As m approaches infinity, this statement simplifies to the traditional Bingham model.

The **yield surface** $y = y_p$ is defined as the location where the local shear stress equals the yield stress, i.e., $|\tau(y_p)| = \tau_0$. For $|\tau| < \tau_0$, the material behaves as a rigid plug region, while for $|\tau| > \tau_0$, it flows as a viscous fluid.

This regularized form was substituted into the non-dimensional momentum equation (17), allowing explicit inclusion of the **Bingham number** $B_n = \tau_0 h/(\mu_c U_0)$. The modified governing equation therefore becomes:

$$Re\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu(T)\frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu(T)}{\partial y}\frac{\partial u}{\partial y} - \left(M^2\sin^2\phi + \frac{\mu(T)}{Da}\right)u + B_n\frac{\partial u}{\partial y} + \sin\phi(G_T T + G_C C) + \frac{Re}{Fr}\sin\varphi.$$

This expression correctly accounts for yield-stress effects and ensures the influence of both yielded and unyielded regions is captured in the flow field. The velocity profiles in Figures 8–24 were reinterpreted under this framework, showing clear flattening within the central plug region, consistent with expected Bingham behaviour.

4. Solution analysis

This part includes analyzing the solutions we obtained. We start with the temperature function Figures 2 and 3, then the concentration function Figures 4–7, and then the fluid velocity function Figures 8–24.

Figure 2 The data indicates that the temperature of the fluid rises as $\mathcal{Q}_H$ and $\mathcal{K}_{\mathcal{T}}$ increase. In contrast, Figure 3 shows that temperature fluid decreases by increasing ω and Pe.

Figures 4–7, Fluid concentration rises with higher values of ω and Pe, but decreases with increased $\mathcal{Q}_H$, $\mathcal{K}_{\mathcal{T}}$, $S_{\mathcal{C}}$, $S_{\mathcal{T}}$, and $\mathcal{K}_{\mathcal{C}}$.

Figures 8–24, Exhibit that fluid velocity escalates with an increase in $\mathcal{Q}_H$, $\mathcal{K}_\mathcal{T}$, $G_\mathcal{C}$, Pe, R_e, $G_\mathcal{T}$, λ,ϕ, φ, and Da, while diminishing with an increase in $S_\mathcal{C}$, $S_\mathcal{T}$, $\mathcal{K}_\mathcal{C}$, M, ω, and F_r. It is evident that the velocity fluctuates more significantly in the scenario of changing viscosity compared to that of constant viscosity.

5. Discussions and conclusion

This study analyzed the transient, incompressible magnetohydrodynamic (MHD) oscillatory flow of a non-Newtonian Bingham fluid through an inclined porous channel, considering variable viscosity, heat generation, and concentration effects. The governing nonlinear equations were solved using the separation of variables technique with computational assistance from Wolfram Mathematica 13. The obtained results were interpreted in terms of the influence of several dimensionless parameters on temperature, concentration, and velocity distributions.

- The velocity of the fluid was observed to increase with larger values of the solutal Grashof number $G_\mathcal{C}$, Reynolds number Re, thermal Grashof number $G_\mathcal{T}$, and pressure gradient constant λ. In contrast, the velocity decreased when the magnetic parameter M, and Froude number Fr were increased. Moreover, the fluid motion was enhanced with a higher channel inclination angle ϕ, as this promotes gravitational acceleration along the channel wall, whereas the effect of the magnetic field inclination φ was relatively minor.

- Increasing the Schmidt number Soret number $S_\mathcal{T}$ and chemical reaction parameter $\mathcal{K}_\mathcal{C}$ resulted in a reduction in concentration, which in turn caused a slight decline in fluid velocity. These trends confirm the coupling between mass diffusion and flow resistance within the porous medium.

- The fluid temperature and velocity rose with larger heat generation $\mathcal{Q}_H$ and radiation $\mathcal{K}_\mathcal{T}$ parameters, as internal heating reduces viscosity and promotes stronger convective motion. Conversely, higher values of the oscillation frequency ω and Péclet number Pe led to a decrease in both temperature and velocity, accompanied by an increase in concentration. This behavior indicates that the influence of temperature on flow acceleration is more dominant than that of concentration.

- The explicit inclusion of the Bingham number (B_n) and the adoption of the Papanastasiou regularization substantially improved the physical fidelity of the model. The revised formulation accurately represents yield-stress effects, capturing both yielded and unyielded (plug) regions within the channel. Increasing $B_n$expands the plug zone near the channel center, while reducing velocity gradients near the walls. These effects become more prominent under higher yield stress or reduced shear conditions, consistent with classical results (Vajravelu et al., 1987; Lakshminarayana et al., 2018). Additionally, it was observed that thermal softening mitigates yield resistance, indicating that elevated temperature partially offsets the retarding influence of yield stress. This refinement enhances the realism of the current Bingham fluid model and aligns it closely with physical flow behavior.

- An increase in the viscosity variation parameter (ε) amplified the velocity magnitude, particularly near the channel center, signifying that thermal dependence of viscosity plays a critical role in controlling the flow field.

- In summary, the present analysis demonstrates that the combined effects of variable viscosity, temperature, concentration, and yield stress govern the dynamic behavior of MHD oscillatory Bingham fluid flow through inclined porous channels. The inclusion of radiation and heat generation parameters provides a more comprehensive understanding of heat and mass transfer mechanisms relevant to industrial, biological, and geophysical transport processes. The extended formulation incorporating the Bingham number and yield-surface representation ensures a more robust and physically accurate model for predicting complex non-Newtonian flow phenomena.

Physical interpretation of the governing effects

The observed flow, thermal, and concentration behaviors can be physically interpreted through the balance between the driving and resisting mechanisms acting in the inclined porous channel. The motion of the fluid is mainly driven by buoyancy forces arising from temperature and concentration differences. These buoyancy effects promote mixed convection and tend to accelerate the flow along the channel. However, this driving mechanism is opposed by several resistance effects, including the Lorentz force due to the applied magnetic field, the drag force caused by the porous medium, the viscous resistance associated with variable viscosity, and the yield-stress resistance of the Bingham fluid.

The magnetic field produces a Lorentz force that acts opposite to the motion of the electrically conducting fluid. Therefore, increasing the magnetic parameter strengthens electromagnetic damping and suppresses the velocity field. In contrast, increasing the thermal or solutal buoyancy contribution enhances the driving force and increases the fluid motion. Hence, the final velocity distribution results from the competition between buoyancy-induced acceleration and magnetic damping.

The Bingham fluid behavior introduces an additional resistance through the yield stress. Since a Bingham fluid does not deform until the applied stress exceeds a critical yield value, increasing the Bingham parameter reduces the ability of the fluid to move freely. This effect becomes more significant when the driving buoyancy force is not strong enough to overcome the yield-stress resistance. Similarly, the porous medium acts as a momentum sink because the solid matrix imposes drag on the moving fluid. As the porous resistance increases, the flow weakens and the velocity decreases.

Variable viscosity further modifies the momentum transport. When viscosity decreases with temperature, the warmer fluid layers become less resistant to shear motion, which enhances the velocity. Conversely, higher viscosity increases the internal friction between adjacent layers and reduces the flow intensity. Therefore, the flow structure is controlled by the combined influence of viscosity variation, yield stress, porous resistance, magnetic damping, and buoyancy forces.

The temperature field is governed by the interaction between thermal diffusion and convective heat transport. When the velocity is enhanced, convective heat transport becomes stronger, allowing heat to be redistributed more effectively through the channel. However, when the magnetic field, porous resistance, or yield stress suppresses the velocity, heat transfer becomes more conduction-dominated. In the same way, the concentration field depends on the balance between molecular diffusion and convective mass transport. Stronger fluid motion enhances species transport, whereas flow suppression causes weaker mass transfer and stronger concentration gradients.

Overall, the results show that the behavior of the system cannot be attributed to a single parameter alone. Instead, the velocity, temperature, and concentration profiles are controlled by the coupled competition between buoyancy forces, Lorentz-force damping, Bingham yield stress, variable viscosity, porous-medium resistance, and thermal and solutal diffusion mechanisms.

Computational procedure and reproducibility

- Software: *Wolfram Mathematica 13*;
- Method: symbolic + numerical integration of the ODE system;
- Mesh size = 500 points, tolerance = 10^{-8}, maximum residual < 10^{-5};
- Convergence criteria and supporting computational materials are provided as extended data in the public repository (see Data Availability section).
- All parameter values used for figures provided.

Data availability

Underlying data

No underlying data are associated with this article. This study is theoretical and computational, and numerical values used to generate all figures are produced dynamically using the Mathematica code provided as extended data.

Extended data

Zenodo: Data and code for: Effects of variable viscosity, concentration and heat variation on MHD oscillatory flow for Bingham fluid through an inclined porous channel.

https://doi.org/10.5281/zenodo.18300187[37]

This project contains the following extended data:

- 1-Velocity.nb – Mathematica code for velocity profiles (case I).
- 2-Velocity.nb – Mathematica code for velocity profiles (case II).
- Temp.nb – Mathematica code for temperature distributions.
- Conce.nb – Mathematica code for concentration distributions.

Data are available under the terms of the Creative Commons Attribution 4.0 International (CC-BY 4.0) license.

Acknowledgments

The authors gratefully acknowledge the support provided by Princess Nourah bint Abdulrahman University through the Researchers Supporting Project number (PNURSP2026R522), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

References

1. Bird RB, Stewart WE, Lightfoot EN: **Transport phenomena.** 1960; **2**.
2. Kapur JN: *Mathematical models in biology and medicine.* Affiliated East-West Press; 1985.
3. Rathy RK: *An Introduction to Fluid Dynamics.* Oxford and IBH; 1976.
4. Vajravelu K, Sreenadh S, Ramakrishna S, *et al.*: **Bingham fluid flow through a circular pipe with permeable wall.** *Z. Angew. Math. Mech.* 1987; **67**: 568–569.
5. Ramakrishna GRS, Sreenadh S, Arunachalam P: **Bingham fluid flow through an inclined channel over a permeable bed.** *Bulletin of Pure and Applied Sciences. E.* 1996; **15**: 47–60.
6. Goverdhan P, Sreenadh S, Arunachalam PV: **Bingham fluid flow between two permeable beds.** *Proc. Nat. Acad. Sci. India.* 1991; **61A**: 19–27.
7. Narahari M, Sreenadh S, Arunachalam PV: **Unsteady flow of a Bingham fluid between two permeable beds.** *J. Indian Acad. Math.* 1999; **22**(2): 225–242.
8. Murthy MR, Prasuna TG, Kumar YR, *et al.*: **Flow of Bingham fluid in contact with a Newtonian fluid.** *Atti della Fondazione Giorgio Ronchi.* 2007; **62**.
9. Tsangaris S, Nikas C, Tsangaris G, *et al.*: **Couette flow of a Bingham plastic in a channel with equally porous parallel walls.** *J. Non-Newtonian Fluid Mech.* 2007; **144**: 42–48.
10. Adnan FA, Hadi AMA: **Effect of an inclined magnetic field on peristaltic flow of Bingham plastic fluid in an inclined symmetric channel with slip conditions.** *Iraqi Journal of Science.* 2019; 1551–1574.
Publisher Full Text
11. Lakshminarayana P, Sreenadh S, Sucharitha G: **The influence of slip, wall properties on the peristaltic transport of a conducting Bingham fluid with heat transfer.** *Procedia Eng.* 2015; **127**: 1087–1094.
12. Mahabaleshwar US, *et al.*: **Effect of MHD and radiation on biviscous Bingham fluid flow on Marangoni boundary for heat source/sink with chemical reaction.** *Case Stud. Therm. Eng.* 2024; **61**: 105105.
Publisher Full Text
13. Basavarajappa M, Myson S, Vajravelu K: **Study of multilayer flow of a bi-viscous Bingham fluid sandwiched between hybrid nanofluid in a vertical slab with nonlinear Boussinesq approximation.** *Phys. Fluids.* 2022; **34**: 12.
Publisher Full Text
14. Afridi M, Chen Z, Qasim M, *et al.*: **Computational analysis of entropy generation minimization and heat transfer enhancement in magnetohydrodynamic oscillatory flow of ferrofluids.** *Journal of Magnetism and Magnetic Materials.* 2024; **594**: 171848.
Publisher Full Text
15. Ullah Z, Makinde O, Hussanan A: **Temperature-dependent density effects on oscillatory mixed convective flow across a non-conducting horizontal circular cylinder embedded in a porous medium.** *International Journal of Ambient Energy.* 2024; **45**(1).
Publisher Full Text
16. Nandkeolyar R, Seth G, Makinde O, *et al.*: **Unsteady Hydromagnetic Natural Convection Flow of a Dusty Fluid Past an Impulsively Moving Vertical Plate With Ramped Temperature in the Presence of Thermal Radiation.** *Journal of Applied Mechanics.* 2013; **80**(6).
Publisher Full Text
17. Vradis GC, Dougher J, Kumar S: **Entrance pipe flow and heat transfer for a Bingham plastic.** *Int. J. Heat Mass Transf.* 1993; **36**: 543–552.
18. Mustafa M, Tabassum M, Rahi M: **Second law analysis of heat transfer in swirling flow of Bingham fluid by a rotating disk subjected to suction effect.** *Therm. Sci.* 2021; **25**: 13–24.
19. Hamza M, Isah B, Usman H: **Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition.** *Int. J. Comput. Appl.* 2011; **33**: 12–17.
20. Khudair WS, Al-Khafajy DGS: **Influence of heat transfer on Magneto hydrodynamics oscillatory flow for Williamson fluid through a porous medium.** *Iraqi Journal of science.* 2018; **59**: 389–397.
21. Al-Aridhee AAH, Al-Khafajy DGS: **Influence of MHD Peristaltic Transport for Jeffrey Fluid with Varying Temperature and Concentration through Porous Medium.** *J. Phys. Conf. Ser.* 2019; **1294**: pp. 032012. IOP Publishing.
Publisher Full Text
22. Al-Khafajy DGS, Labban JA: **Temperature and concentration effects on oscillatory flow for variable viscosity carreau fluid through an inclined porous channel.** *Iraqi Journal of Science.* 2021; 45–53.
Publisher Full Text
23. Liu C, Pan M, Zheng L, *et al.*: **Flow and heat transfer of Bingham plastic fluid over a rotating disk with variable thickness.** *Zeitschrift Für Naturforschung A.* 2016; **71**: 1003–1015.
24. Eldabe NT, Abouzeid M, Shawky HA: **MHD peristaltic transport of Bingham blood fluid with heat and mass transfer through a non-uniform channel.** *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences.* 2021; **77**: 145–159.
25. Al-Khafajy DGS, Mohammed SN: **Influence of Heat Transfer on Magnetohydrodynamics Oscillatory Flow for Bingham Fluid with Variable Viscosity Through a Porous channel.** *J. Phys. Conf. Ser.* 2021; **1999**: pp. 012104. IOP Publishing.
Publisher Full Text
26. Salahuddin T, Awais M, Muhammad S: **Featuring the aspects with temperature dependent viscosity of inclined MHD Williamson fluid along with heat source/sink, Soret and Dufour effects: A predictor-corrector approach.** *International Communications in Heat and Mass Transfer.* 2024; **159**: 108178.
Publisher Full Text
27. Akram S, Nadeem S, Hussain A: **Effects of heat and mass transfer on peristaltic flow of a Bingham fluid in the presence of inclined magnetic field and channel with different wave forms.** *J. Magn. Magn. Mater.* 2014; **362**: 184–192.
28. Humnekar N, Darbhasayanam S: **The stability of the nanofluid flow in an inclined porous channel with variable viscosity.** *Numer. Heat Transf. A Appl.* 2023; 1–14.
29. Panda S, Raizah Z, Mishra S, *et al.*: **Numerical analysis of Marangoni convection in a tangent hyperbolic fluid with Cattaneochristov heat and mass flux under buoyancy and activation energy effects.** *Results in Engineering.* 2026; **29**: 108548.
Publisher Full Text
30. Sneha K, Bognar G, Mahabaleshwar U, *et al.*: **Magnetohydrodynamics effect of Marangoni nano boundary layer flow and heat transfer with CNT and radiation.** *Journal of Magnetism and Magnetic Materials.* 2023; **575**: 170721.
Publisher Full Text
31. Sachhin S, Mahabaleshwar U, Bognar G, *et al.*: **Rheological analysis of thermodynamics and viscosity ratio impact on non-Newtonian tetra-Bingham nanofluid stagnation point flow driven by stretching plate and a circular cylinder.** *International Journal of Numerical Methods for Heat & Fluid Flow.* 2026; **36**(1): 254–274.
Publisher Full Text
32. Lakshminarayana P, Vajravelu K, Sucharitha G, *et al.*: **Peristaltic slip flow of a Bingham fluid in an inclined porous conduit with Joule**

heating. *Applied Mathematics and Nonlinear Sciences.* 2018; **3**: 41–54.
Publisher Full Text

33. Mohammed SN, Al-Khafajy DGS: **Influence of Temperature and Concentration on MHD Oscillatory Flow for Bingham Fluid with Variable Viscosity Through an inclined channel.** *Al-Qadisiyah Journal of Pure Science.* 2021; **26**: 324–346.

34. Kumbinarasaiah S: **Entropy generation on an MHD Casson fluid flow in an inclined channel with a permeable walls through Hermite wavelet method.** *Results in Control and Optimization.* 2023; **12**: 100261.

35. Jha BK, Aina B: **MHD mixed convection flow in an inclined porous channel having time-periodic boundary condition (MHD mixed convection flow in an inclined porous channel).** *Journal of Porous Media.* 2021; **24**: 69–91.
Publisher Full Text

36. Al-Khafajy DGS: **Radiation and mass transfer effects on MHD oscillatory flow for carreau fluid through an Inclined porous channel.** *Iraqi Journal of science.* 2020; 1426–1432.
Publisher Full Text

37. Al-Khafajy D, Al-Hanaya A, Alotaibi MA: **Effects of variable viscosity, concentration and heat variation on MHD oscillatory flow for Bingham fluid through an inclined porous channel.** *Zenodo.* Jan. 19, 2026.
Publisher Full Text

Open Peer Review

Current Peer Review Status: ✔ ? ✔ ?

Version 2

Reviewer Report 11 July 2026

https://doi.org/10.5256/f1000research.201751.r496492

© **2026 P L.** This is an open access peer review report distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

?

Lakshminarayana P
Vellore Institute of Technology, Vellore, Tamil Nadu, India

This study investigates the effects of variable viscosity, concentration, and heat transfer on the magnetohydrodynamic (MHD) oscillatory flow of a Bingham fluid through an inclined porous channel. The influence of key physical parameters on the velocity, temperature, and concentration distributions is analyzed to better understand the transport characteristics of non-Newtonian fluids in porous media. Below, I have outlined some constructive feedback and recommendations to improve the manuscript.

1. The abstract does not mention the practical applications of the present study. The authors are requested to include the engineering and/or real-world applications of the proposed model in the abstract.
2. The introduction relies on several older references. It is recommended to incorporate more recent and relevant literature to better reflect the current state of research.
3. Appropriate references should be provided for the governing equations to establish the validity and reliability of the mathematical formulation.
4. The manuscript should clearly specify whether the porous medium is distributed throughout the channel or is confined to the channel walls.
5. How this study is different from the following recent studies. Authors may include the discussion in the introduction to strengthen the background on magnetohydrodynamics, inclined channels, and buoyancy effects: Magnetohydrodynamic Peristaltic Flow of Hybrid Nanofluid in an Asymmetric Channel with Thermal Radiation and Shape Factors: An Application to Coolant Systems; Convective peristaltic pumping of MHD Ree-Eyring nanofluid in a chemically reacting flexible divergent channel with activation energy and radiation; Investigation of a conducting Casson fluid flow through a porous flexible microfluidic channel with catalytic effects: application in pharmaceutical fluid processing; Cilia-Mediated Electroosmotic Magnetohydrodynamic Hybrid Nanofluid Transport for Biomedical Applications: Impact of Radiation and Nanoparticle Geometry.
6. To better highlight the novelty of the present work, a comparison (novelty) table should be included at the end of the introduction, clearly distinguishing the proposed study from previously published works.
7. The figures should include directional arrows indicating the influence (increasing or

decreasing trends) of the governing parameters on the flow characteristics to improve clarity and readability.
8. The basis for selecting the ranges of the governing parameters should be clearly justified, preferably with appropriate references or practical considerations.
9. The author (s) should give a proper discussion for every figure presented in the results and discussion section, along with the physical mechanism. Use the arrow in the figures for a better understanding of the trend of the given parameter on the flow. Please see the refs: Diffusion effects on mixed convective peristaltic flow of a bi-viscous Bingham nanofluid through a porous medium with convective boundary conditions; Energy Diffusion Efficiency through Peristaltic–Electroosmotic Flow of (Cu–TiO_2–GO)/H_2O Trihybrid Nanofluid in Sterilization and Pasteurization: Application to Food Processing; Bioadaptive peristaltic model of double-diffusive nanofluid through a slant porous channel: application to hazardous compound mitigation;
10. A detailed physical interpretation should be provided for the effects of all governing parameters discussed in the results section.
11. The manuscript should present separate sections for Results and Discussion and Conclusion to improve its organization and readability.
12. A dedicated discussion on the limitations of the present study and possible directions for future research should be included.
13. The numerical or analytical results should be validated through comparison with previously published results or established benchmark solutions.
14. A comprehensive nomenclature section should be included, listing all symbols, abbreviations, and their corresponding SI units.

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and is the work technically sound?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Partly

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Fluid dynamics; Heat transfer, MHD; Boundary layer theory; Peristalsis; Flow through Porous media.

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 10 July 2026

https://doi.org/10.5256/f1000research.201751.r496483

© **2026 Chamkha A.** This is an open access peer review report distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Ali J. Chamkha
Kuwait College of Science and Technology, Doha, Kuwait

The paper is a revised version that this reviewer did not review before. The paper is good but can be improved. Therefore, the paper can be recommended for indexing provided that the authors address the following comments:

1. Add a Nomenclature.
2. The paper can benefit from editing the English language throughout.
3. The literature review is good but can be improved. The following related work can be checked and considered: Hydromagnetic natural convection from an inclined porous square enclosure with heat generation.
4. Discuss a real application for this problem with the many included effects.
5. Justify why the convective terms in the equations are neglected.
6. Based on the order of magnitudes analysis, explain why the magnetic, porous resistance and the viscous dissipations are not included in this study.
7. Justify the neglection of the induced magnetic field and Hall effect. See for instance, the following related work: Hall effects on mhd peristaltic flow of Jeffrey fluid through porous medium in a vertical stratum; Unsteady flow and heat transfer on a semi-infinite flat plate with an aligned magnetic field.
8. State the condition for which the Forchheimer inertia effect of the porous media becomes important and justify its neglection. See for example, Darcy-Forchheimer relation in Casson type MHD nanofluid flow over non-linear stretching surface.
9. Justify the assumption of local thermal equilibrium between the fluid and the porous media in one region.
10. Why is the porosity of the media not included in the equations? Please explain.
11. Comparisons with previously published work should be made to ensure correctness of the closed-form solutions.
12. On what basis are the parametric values chosen? Do they correspond to a specific physical condition?

Is the work clearly and accurately presented and does it cite the current literature?
Yes

Is the study design appropriate and is the work technically sound?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Thermo-fluid Sciences

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Reviewer Report 01 July 2026

https://doi.org/10.5256/f1000research.201751.r496026

© 2026 Makinde O. This is an open access peer review report distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Oluwole Daniel Makinde
Stellenbosch University, Cape Town, South Africa

The revised version seems okay.

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Fluid Mechanics, Thermal Sciences, MHD, Mathematical modelling, Heat and Mass Transfer

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Version 1

Reviewer Report 07 May 2026

https://doi.org/10.5256/f1000research.190674.r473533

© **2026 Bognár G.** This is an open access peer review report distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

?

Gabriella Bognár
University of Miskolc, Miskolc-Egyetemváros, Hungary

The reviewer already identified a major inconsistency that the momentum equation and dimensionless equation do not match. This is a fundamental problem. Correct governing equations.

Please highlight the novel aspects of the article in the abstract and at the end of the introduction.

The studied effects the MHD flow, Bingham fluid, variable viscosity, concentration / heat transfer and porous medium are all well studied in literature, many cited papers cover similar formulations.

The physical interpretation of the results are mostly “velocity increases with parameter X”, “temperature decreases with parameter Y”. It lacks the force balance interpretation and the competition of effects of Lorentz force vs buoyancy and yield stress vs viscosity.

The paper needs language polishing!

Some parameters not physically explained.

The literature is mainly descriptive. Update the introduction section. Remove some old-fashioned studies. An updated and complete literature review should be conducted. It should appear as part of the Introduction while bearing in mind the work's taking into account the scope and the readership of the journal.

You can consider the following:
Reference 1
Reference 2
Reference 3

Add a comparison with literature.

References

1. Panda S, Raizah Z, Mishra S, Baithalu R, et al.: Numerical analysis of Marangoni convection in a tangent hyperbolic fluid with Cattaneochristov heat and mass flux under buoyancy and activation energy effects. *Results in Engineering*. 2026; **29**. Publisher Full Text
2. Sneha K, Bognar G, Mahabaleshwar U, Singh D, et al.: Magnetohydrodynamics effect of Marangoni nano boundary layer flow and heat transfer with CNT and radiation. *Journal of*

Magnetism and Magnetic Materials. 2023; **575**. Publisher Full Text
3. Sachhin S, Mahabaleshwar U, Bognar G, Huang H, et al.: Rheological analysis of thermodynamics and viscosity ratio impact on non-Newtonian tetra-Bingham nanofluid stagnation point flow driven by stretching plate and a circular cylinder. *International Journal of Numerical Methods for Heat & Fluid Flow*. 2026; **36** (1): 254-274 Publisher Full Text

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and is the work technically sound?
Partly

Are sufficient details of methods and analysis provided to allow replication by others?
No

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Heat and mass transfer of nanofluid flows, tribology

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Author Response 15 May 2026

Munirah Alotaibi

Response to Reviewer 2

Thank you very much for your careful evaluation of our manuscript and for your constructive comments. We have revised the manuscript thoroughly according to the points raised.

Comment 1: The reviewer already identified a major inconsistency: the momentum equation and dimensionless equation do not match. This is a fundamental problem. Correct governing equations.

Response: We sincerely thank the reviewer for pointing out this important issue. We

agree that consistency between the dimensional and dimensionless governing equations is essential. Therefore, the governing momentum equation has been carefully rechecked and corrected. The nondimensionalization procedure has also been revised to ensure that the associated dimensionless equation follows correctly from the dimensional model. The related equations, notation, and parameter definitions have been checked throughout the manuscript to avoid any further inconsistency.

Comment 2: Please highlight the novel aspects of the article in the abstract and at the end of the Introduction.

Response: Thank you for this valuable suggestion. The abstract has been revised to highlight the novelty of the study more clearly. In addition, a novelty statement has been added at the end of the Introduction. The revised text now emphasizes that the contribution of the present work lies in developing an analytical formulation for oscillatory MHD mixed convection flow of a Bingham fluid through an inclined porous channel while considering the combined effects of variable viscosity, porous-medium resistance, heat generation, radiation, concentration transport, magnetic damping, and yield-stress behavior.

Comment 3: The studied effects, including MHD flow, Bingham fluid, variable viscosity, concentration/heat transfer, and porous medium, are all well studied in the literature, and many cited papers cover similar formulations.

Response: We agree with the reviewer that previous studies have widely investigated these individual effects. Accordingly, the revised manuscript now clarifies that the novelty is not based on each effect individually but on their combined treatment within a single analytical oscillatory channel-flow model. The Introduction has been improved to better identify the research gap and to explain how the present formulation differs from earlier studies that considered these mechanisms separately or in different flow configurations.

Comment 4: The physical interpretation of the results is mostly limited to statements such as “velocity increases with parameter X” and “temperature decreases with parameter Y.” It lacks force-balance interpretation and the competition of effects of Lorentz force versus buoyancy and yield stress versus viscosity.

Response: Thank you for this important observation. The physical interpretation of the results has been strengthened in the revised manuscript. A dedicated explanation has been added to clarify the competition between the driving buoyancy forces and the resisting Lorentz force. The discussion also explains the influence of Bingham yield stress, viscosity variation, and porous-medium drag on the flow behavior. This revision provides a more mechanistic interpretation rather than only describing whether the velocity, temperature, or concentration increases or decreases with a given parameter.

Comment 5: The paper needs language polishing.

Response: The manuscript has been carefully revised and language-polished to improve grammar, readability, sentence structure, and scientific clarity.

Comment 6: The literature is mainly descriptive. Update the Introduction section. Remove some old-fashioned studies. An updated and complete literature review should be conducted. It should appear as part of the introduction while bearing in mind the scope and readership of the journal.

Response: Thank you for this helpful suggestion. The introduction has been revised to improve the literature review and make it more focused on the scope and readership of the journal. Additional recent and relevant studies have been incorporated, including the references suggested by the reviewer. We have not removed the older references because they provide useful background for the development of the topic; however, the revised introduction now better connects both earlier and recent studies to the present problem and clearly identifies the remaining research gap. We have also reduced the reliance on purely descriptive statements by linking previous studies more directly to the motivation and novelty of the present work.

Comment 7: You can consider the following references: Reference 1, Reference 2, and Reference 3.

Response: Thank you for suggesting these relevant studies. The recommended references have been added to the revised introduction. They have been used to strengthen the discussion of recent developments in non-Newtonian fluid flow, MHD transport, thermal radiation, rheological effects, and heat and mass transfer mechanisms.

We sincerely thank the reviewer for these valuable comments, which have helped us improve the mathematical consistency, novelty presentation, literature background, physical interpretation, parameter explanation, and overall clarity of the manuscript.

Competing Interests: No competing interests were disclosed.

Reviewer Report 28 April 2026

https://doi.org/10.5256/f1000research.190674.r473534

© **2026 Makinde O.** This is an open access peer review report distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

?

Oluwole Daniel Makinde
Stellenbosch University, Cape Town, South Africa

The authors presented an analytical solution to a model problem that described MHD mixed convection oscillatory flow of Bingham fluid through an inclined porous channel. The following improvement must be incorporated:
1.) Include appropriate expressions for the skin friction, Nusselt number, and Sherwood number.

Figures showing the effects of emerging parameters on skin friction, Nusselt number, and Sherwood number must be incorporated.
2.) Include figures depicting the impact of the oscillatory nature on the velocity, temperature, and concentration profiles.
3.) The model momentum equation (13) failed to reflect the associated dimensionless equation (17). Correct this major technical error and rework the entire manuscript.
4.) Update the writeup with the following relevant published papers:
Reference 1. *Journal of Magnetism and Magnetic Materials, Vol. 594, 171848(17pages), 2024.*
Reference 2. Journal of Applied Mechanics, Vol. 80(6), 061003, 2013.
Reference 3 *International Journal of Ambient Energy, Vol.45(1), 2423226 (8pages), 2024.*

References

1. Afridi M, Chen Z, Qasim M, Makinde O: Computational analysis of entropy generation minimization and heat transfer enhancement in magnetohydrodynamic oscillatory flow of ferrofluids. *Journal of Magnetism and Magnetic Materials*. 2024; **594**. Publisher Full Text
2. Nandkeolyar R, Seth G, Makinde O, Sibanda P, et al.: Unsteady Hydromagnetic Natural Convection Flow of a Dusty Fluid Past an Impulsively Moving Vertical Plate With Ramped Temperature in the Presence of Thermal Radiation. *Journal of Applied Mechanics*. 2013; **80** (6). Publisher Full Text
3. Ullah Z, Makinde O, Hussanan A: Temperature-dependent density effects on oscillatory mixed convective flow across a non-conducting horizontal circular cylinder embedded in a porous medium. *International Journal of Ambient Energy*. 2024; **45** (1). Publisher Full Text

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and is the work technically sound?
Partly

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Fluid Mechanics, Thermal Sciences, MHD, Mathematical modelling, Heat and Mass Transfer

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Author Response 15 May 2026

Munirah Alotaibi

Thank you very much for your careful reading of the manuscript and for your valuable comments and suggestions. We have revised the manuscript carefully in response to the points raised.

Comment 1: Include appropriate expressions for the skin friction, Nusselt number, and Sherwood number. Figures showing the effects of emerging parameters on skin friction, Nusselt number, and Sherwood number must be incorporated.

Response: Thank you for this important suggestion. We fully acknowledge that the skin friction coefficient, Nusselt number, and Sherwood number are useful engineering quantities for evaluating momentum, heat-transfer, and mass-transfer rates. However, the main objective of the present work is to obtain and discuss the analytical solutions for the velocity, temperature, and concentration distributions of MHD mixed convection oscillatory flow of a Bingham fluid through an inclined porous channel. Therefore, in the revised manuscript, we have kept the focus on the analytical field solutions and their physical interpretation. Adding separate expressions and additional graphical analyses for skin friction, Nusselt number, and Sherwood number would substantially extend the scope of the present study. Accordingly, these quantities are not included in the current revision and may be considered in a future extension of this work.

Comment 2: Include figures depicting the impact of the oscillatory nature on the velocity, temperature, and concentration profiles.

Response: Thank you for this constructive comment. The oscillatory nature of the flow has already been examined in the manuscript through the oscillation frequency parameter (\omega). Figures showing the effects of different values of (\omega) on the velocity, temperature, and concentration profiles are included (Figs. 3, 4, 22, 23).

Comment 3: The model momentum equation (13) failed to reflect the associated dimensionless equation (17). Correct this major technical error and rework the entire manuscript.

Response: We sincerely thank the reviewer for identifying this important issue. We agree that the inconsistency between the dimensional momentum equation and the corresponding dimensionless equation was a typographical error. The model momentum equation has now been corrected, and the nondimensionalization procedure has been carefully rechecked. The related equations, parameter definitions, notation, and discussion have also been revised to ensure consistency between the dimensional and dimensionless formulations throughout the manuscript.

Comment 4: Update the write-up with the following relevant published papers:
Reference 1. Journal of Magnetism and Magnetic Materials, Vol. 594, 171848, 2024.
Reference 2. Journal of Applied Mechanics, Vol. 80(6), 061003, 2013.
Reference 3. International Journal of Ambient Energy, Vol. 45(1), 2423226, 2024.

Response: Thank you for suggesting these relevant references. The introduction has been updated by incorporating the recommended studies. These papers have been used to strengthen the literature background related to MHD oscillatory flow, unsteady hydromagnetic convection, porous media, heat transfer, and mixed convection. The revised introduction now better reflects recent developments and places the present work more clearly within the existing literature.

We sincerely appreciate the reviewer's constructive comments, which helped us improve the mathematical consistency, clarity, and presentation of the manuscript.

Competing Interests: No competing interests were disclosed.

The benefits of publishing with F1000Research:

- Your article is published within days, with no editorial bias
- You can publish traditional articles, null/negative results, case reports, data notes and more
- The peer review process is transparent and collaborative
- Your article is indexed in PubMed after passing peer review
- Dedicated customer support at every stage

For pre-submission enquiries, contact research@f1000.com

F1000Research